UG-5/96
HUB-EP-96/15
hep-th/9605216
May 1996

A Note on Intersecting D-branes and Black Hole Entropy

Klaus Behrndt

Humboldt-Universität, Institut für Physik
Invalidenstraße 110, 10115 Berlin
Germany

and

Eric Bergshoeff

Institute for Theoretical Physics
Nijenborgh 4, 9747 AG Groningen
The Netherlands

Abstract

In four dimensions there are 4 different types of extremal Maxwell/scalar black holes characterized by a scalar coupling parameter a with $a = 0, 1/\sqrt{3}, 1, \sqrt{3}$. These black holes can be described as intersections of ten–dimensional non-singular Ramond-Ramond objects, i.e. D-branes, waves and Taub-NUT solitons. Using this description it can be shown that the four–dimensional black holes decompactify near the core to higher–dimensional *non-singular* solutions. In terms of these higher–dimensional non-singular solutions we define a non-vanishing entropy for all four black hole types from a four–dimensional point of view.

1. Introduction

A common way to classify four–dimensional Maxwell/scalar black hole (BH) solutions is to specify the coupling of the scalar fields to the gauge fields. In the simplest case of only one scalar field and one gauge field this coupling is characterized by a single parameter a and the action in the Einstein frame is given by

$$S_{4d} = \frac{1}{16\pi G_4} \int d^4x \sqrt{|g|} \{-R + 2(\partial\phi)^2 + e^{-2a\phi} F^2\} \ , \tag{1}$$

where G_4 is the 4–dimensional Newton constant. There exists four different types of extremal[1] black hole solutions, which are defined in terms of a function $H(\vec{x})$ which is harmonic on the 3–dimensional transverse space. The metric of these solutions is given by

$$\begin{array}{llll} a = 0 & : \ ds^2 = H^{-2}dt^2 - H^2 d\vec{x}^2 & , & e^{-2\phi} = 1 \ , \\ a = 1/\sqrt{3} & : \ ds^2 = H^{-3/2}dt^2 - H^{3/2} d\vec{x}^2 & , & e^{\pm 2\phi/\sqrt{3}} = \sqrt{H} \ , \\ a = 1 & : \ ds^2 = H^{-1}dt^2 - H d\vec{x}^2 & , & e^{\pm 2\phi} = H \ , \\ a = \sqrt{3} & : \ ds^2 = H^{-1/2}dt^2 - H^{1/2} d\vec{x}^2 & , & e^{\pm 2\phi/\sqrt{3}} = \sqrt{H} \ . \end{array} \tag{2}$$

The harmonic function $H(\vec{x})$ is given by

$$H(\vec{x}) = 1 + \frac{r_h}{r} \ , \tag{3}$$

where $r^2 = \vec{x}\cdot\vec{x}$ and r_h is proportional to the charge. These solutions have been generalized to different harmonic functions in [1] (for $a = 0$ this generalization has been given in [2]). For a recent discussion of these solutions as bound states, see [3]. The four solutions (2) are also known as

$a = 0$: 4d Reissner-Nordstom (RN) solution,

$a = 1/\sqrt{3}$: 5d RN (ϕ is a modulus field),

$a = 1$: dilaton black hole (ϕ has standard dilaton coupling),

$a = \sqrt{3}$: 5d KK black hole.

For $a \neq 0$ the gauge fields can be electric or magnetic. The two possibilities correspond to different signs of the scalar field ϕ. In formula (2) the "+" sign corresponds to the magnetic case. On the other hand, the $a = 0$ RN solution is dyonic. It turns out that in four dimensions only the $a = 0$ RN solution is non-singular at the horizon $r = 0$. However, following [4], also the $a \neq 0$ solutions can be understood in a non-singular way, in the sense that they follow from the dimensional reduction of the following higher–dimensional non-singular solutions:

[1] In this letter we consider only extremal solutions.

$$a = 1/\sqrt{3} \quad : \quad \text{5d RN electric black hole or magnetic string} \ ,$$
$$a = 1 \quad : \quad \text{6d self-dual string} \ ,$$
$$a = \sqrt{3} \quad : \quad \text{10d self-dual } D\text{-3-brane} \ .$$

All solutions can be understood as intersections of branes in 10 or 11 dimensions [6, 7, 8, 9, 10, 11]. Most of them are singular. But there are some objects which are non-singular. In ten dimensions these are: D-3-branes, gravitational waves and Taub-NUT solitons[2]. Only the first object carries Ramond–Ramond (RR) charges and is part of the D-branes. The wave and Taub-NUT solitons are neutral and do not fit in the standard brane picture. They appear as the T-dual of the fundamental NS string and NS 5-brane. Since they are solutions of pure gravity (without scalars and gauge fields) we can regard them as solutions of the RR sector. Although they are neutral in 10 dimensions their metric is non-diagonal and yields KK gauge fields in lower dimensions. The aim of this letter is to understand the black holes (2) and their higher–dimensional origins in terms of an intersection of these ten–dimensional non-singular objects. In analogy to [1, 3] we can see this orthogonal intersection as bound states of fundamental $a = \sqrt{3}$ states (a single object or brane).

2. Black Holes as intersecting D–branes

Since the metric of a single D–brane solution in ten dimension involves the square root of a harmonic function and the power of the harmonics in (2) is at most two we know that in order to describe the BH's in (2) as intersections we need at most 4 objects in 10 dimensions, each defined by its own harmonic function. At this point one could ask whether there exist more BH's, e.g. described by 5,6,.. harmonic functions. It is easy to see that this is not possible. An odd number of harmonic functions is ruled out by our restriction to find a non-singular intersection, we need an even number to keep the compactification radii finite. But why not 6 harmonic functions? A further restriction is that for 2 non-trivial functions only the self-dual string describes a non-singular intersection, i.e. any pair of harmonics has to describe this string. For 4 objects we can build 6 pairs and the corresponding strings fit nicely in the 6 internal directions. On the other side for 6 objects we can build 15 pairs or strings and there is no way to put these strings into the internal space. Some of them have to lie in the same internal direction resulting in a singular scalar field. With similar arguments one can discard also the higher cases. Thus let us come back to the case of 4 intersecting objects. For the $a = 0$ case all functions are non-trivial. Since this case is dyonic there are two possible intersections: $(\mathit{3} \times \mathit{3} \times \mathit{3} \times \mathit{3})$ where all objects are dyonic or $(\mathit{3} \times \mathit{3} \times \tilde{\mathit{1}} \times \tilde{\mathit{5}})$ where the electric (KK) charge coming from the wave $(= \tilde{\mathit{1}})$

[2] We do not discuss the 2- and 5-brane solutions of the 11–dimensional M-theory, which are also non-singular. They appear in 10 dimensions as a subset of the D-branes which we prefer to consider. There are also other branes that have a non-singular metric , e.g. the $p = -1$ brane which is a wormhole in the string frame [5]. However, these solutions are not asymptotically flat and/or have a singular dilaton. We will ignore these solutions as well.

compensates the magnetic charge from the Taub-NUT soliton ($= \tilde{5}$)[3]. We consider these two cases separately.

(i) the $(3 \times 3 \times 3 \times 3)$ **case**

We take 4 intersecting D-3-branes with metric given by [7, 8]

$$\begin{aligned} ds^2 &= \frac{1}{\sqrt{H_1H_2H_3H_4}}dt^2 - \sqrt{H_1H_2H_3H_4}d\vec{x}^2 - \sqrt{\frac{H_1H_2}{H_3H_4}}dx_4^2 - \sqrt{\frac{H_1H_3}{H_2H_4}}dx_5^2 - \\ &\quad -\sqrt{\frac{H_1H_4}{H_2H_3}}dx_6^2 - \sqrt{\frac{H_2H_3}{H_1H_4}}dx_7^2 - \sqrt{\frac{H_2H_4}{H_1H_3}}dx_8^2 - \sqrt{\frac{H_3H_4}{H_1H_2}}dx_9^2 \,. \end{aligned} \tag{4}$$

The electric gauge field components are

$$\begin{aligned} F \sim \ & dt \wedge \Big(dH_1^{-1} \wedge dx_7 \wedge dx_8 \wedge dx_9 + dH_2^{-1} \wedge dx_5 \wedge dx_6 \wedge dx_9 + \\ & + dH_3^{-1} \wedge dx_4 \wedge dx_6 \wedge dx_8 + dH_4^{-1} \wedge dx_4 \wedge dx_5 \wedge dx_7\Big) \ . \end{aligned} \tag{5}$$

The magnetic components can be obtained by using the self-duality condition in $D = 10$. The special cases (2) appear in the limit

$$\begin{aligned} a = 0 \ &: \ H_1 = H_2 = H_3 = H_4 = H \ , \\ a = 1/\sqrt{3} \ &: \ H_1 = H_2 = H_3 = H \quad , \quad H_4 = 1 \ , \\ a = 1 \ &: \ H_1 = H_2 = H \quad , \quad H_3 = H_4 = 1 \ , \\ a = \sqrt{3} \ &: \ H_1 = H \quad , \quad H_2 = H_3 = H_4 = 1 \ . \end{aligned} \tag{6}$$

The harmonic function H depends on all overall transversal coordinates, i.e. the coordinates for which the metric has no H in the denominator. The last case, e.g. , is the single 3-brane with a harmonic function depending on: $\vec{x}, x_4, x_5, x_6$ and for the $a = 1$ case H is harmonic with respect to the coordinates: $\vec{x}, x_4$. The solution (4) is non-singular for the case $a = 0, 1, \sqrt{3}$. However, the case $a = 1/\sqrt{3}$ is singular. We next consider the second intersection.

(ii) the $(3 \times 3 \times \tilde{1} \times \tilde{5})$ **case**

We intersect two D-3-branes with a wave and a Taub-NUT soliton. The resulting metric is given by

$$\begin{aligned} ds^2 &= \frac{1}{\sqrt{H_1H_2}}du(dv - \tilde{H}_1 du) - \sqrt{H_1H_2}\left[\frac{1}{\tilde{H}_5}(dx_5 + \vec{V}d\vec{x})^2 + \tilde{H}_5 d\vec{x}^2\right] - \\ &\quad -\sqrt{\frac{H_1}{H_2}}\,(dx_6^2 + dx_7^2) - \sqrt{\frac{H_2}{H_1}}\,(dx_8^2 + dx_9^2) \ . \end{aligned} \tag{7}$$

[3]We do not consider the case $(\tilde{1} \times \tilde{5} \times \tilde{1} \times \tilde{5})$ where none of the components exhibits a horizon. We denote the wave with $\tilde{1}$ to indicate that it is T-dual to the fundamental string *1* and the Taub-NUT soliton with $\tilde{5}$ since it is T-dual to the solitonic 5-brane *5*.

The electric part of the field strength is given by

$$F \sim dv \wedge du \wedge \Big(dH_1^{-1} \wedge dx_8 \wedge dx_9 + dH_2^{-1} \wedge dx_6 \wedge dx_7 \Big) \ , \tag{8}$$

where $v = t + x_4$, $u = t - x_4$ and $\vec{\nabla}\tilde{H}_5 = \vec{\nabla} \times \vec{V}$. This is a non-diagonal intersection yielding in 4 dimensions two D-brane or RR charges and two KK charges. Compactifying this model to 6 dimensions and performing a type IIB $SL(2,R)$ transformation this model coincides with the solution discussed in [2], which is self-dual under T–duality as well as string/string duality. In 10 dimensions the wave lying on the common world volume of the 3-branes and the Taub-NUT soliton in the overall transversal space correspond to additional momentum modes in the internal space. Again we get the 4 BH's (2) if we choose the harmonic functions properly:

$$\begin{array}{rcll}
a = 0 & : & H_1 = H_2 = \tilde{H}_1 = \tilde{H}_5 = H \ , & \\
a = 1/\sqrt{3} & : & H_1 = H_2 = \tilde{H}_1 = H \quad , \quad \tilde{H}_5 = 1 & \text{(electric)}, \\
 & & H_1 = H_2 = \tilde{H}_5 = H \quad , \quad \tilde{H}_1 = 1 & \text{(magnetic)}, \\
a = 1 & : & H_1 = H_2 = H \quad , \quad \tilde{H}_1 = \tilde{H}_5 = 1 \ , & \\
a = \sqrt{3} & : & H_1 = H \quad , \quad \tilde{H}_1 = \tilde{H}_5 = H_2 = 1 \ . &
\end{array} \tag{9}$$

As before, we assume that the harmonic functions depend only on the overall transversal coordinates. The last two cases in (9) describe the same intersections as before. In contrast to the intersection of 4 D-3-branes, all cases, including the $a = 1/\sqrt{3}$ case, are non-singular if we approach the horizon at $r = 0$.

3. Entropy

Our aim is to use the higher–dimensional interpretation in terms of intersecting D–branes to discuss the entropy of the four–dimensional black holes. To explain the main idea we first consider the electric $a = 1/\sqrt{3}$ solution in more detail. In this case after a trivial reduction (yielding no KK scalars) the 5d solution is

$$ds^2 = \frac{1}{H^2} dt^2 - H(dx_5^2 + d\vec{x}^2) \quad , \quad F_{0m} \sim \partial_m H^{-1} \, , \tag{10}$$

where $x_m = (x_5, \vec{x})$ and

$$H(x_5, \vec{x}) = 1 + \frac{r_h^2}{\rho^2} \, , \tag{11}$$

with $\rho^2 = x_5^2 + r^2$. This is the electric 5d RN solution. The standard $a = 1/\sqrt{3}$ BH is obtained after compactification over x_5. Hence, we have to assume that H is periodic: $x_5 \sim x_5 + 2\pi R$. Then we can make the standard ansatz for H as a periodic array [12]

$$H = 1 + \sum_{n=-\infty}^{+\infty} \frac{r_h^2}{r^2 + (x_5 + 2\pi n R)^2} = 1 + \frac{r_h^2}{2R\, r} + \mathcal{O}(e^{-\frac{r}{R}}) \ . \tag{12}$$

Thus, away from the origin the dependence on x_5 is exponentially suppressed, this direction is compactified on a circle with radius R. On the other side near the horizon ($\rho = 0$) one can "feel" the x_5 dependence and the solution decompactifies to its 5d origin. The philosophy is the same as for the example of rotating BH's discussed in [13]. The asymptotic behaviour of the metric near the horizon is given by

$$ds^2 \to \left(\frac{\rho^2}{r_h^2}\right)^2 dt^2 - r_h^2 \left(\frac{d\rho}{\rho}\right)^2 - r_h^2 d\Omega_3^2 = e^{4\eta/r_h} dt^2 - d\eta^2 - r_h^2 d\Omega_3^2 \,, \tag{13}$$

where $\rho/r_h = e^{\eta/r_h}$ and r_h is the radius of the S_3 sphere. Hence, the asymptotic geometry is: (de Sitter)$_2 \times S_3$ which is non-singular. In analogy to this procedure one finds for the other cases [4]:

$$\begin{aligned}
a = 0 \quad &: \quad (\mathrm{AdS})_2 \times S_2 \ , \\
a = 1/\sqrt{3} \quad &: \quad (\mathrm{AdS})_2 \times S_3 \qquad \text{(5d electric RN BH)}, \\
& \quad (\mathrm{AdS})_3 \times S_2 \qquad \text{(5d magnetic RN string)}, \\
a = 1 \quad &: \quad (\mathrm{AdS})_3 \times S_3 \qquad \text{(6d self-dual string)}, \\
a = \sqrt{3} \quad &: \quad (\mathrm{AdS})_5 \times S_5 \qquad \text{(10d self-dual 3-brane)} \,,
\end{aligned} \tag{14}$$

where (AdS)$_n$ is the n-dimensional anti de Sitter space. These asymptotic limits arise also in the extreme limit of non-extremal black p-branes [14]. In [4] it has been shown that it is possible to extend the solutions through the horizon. As a result it was found that for the 4d ($a = 0$) and the 5d electric RN BH there is an interior region with a curvature singularity. The other cases, however, are completely singularity free, i.e. they describe a space time without any singularities but with a horizon.

Our purpose is to use the description of the BH's given in (2) as intersections to define an entropy for all BH's in (2). For every black hole solution one defines the entropy via the Bekenstein-Hawking formula

$$S = \frac{A}{4} \ , \tag{15}$$

where A is the area of the horizon ($r = 0$) and we set the 4d Newton constant $G_4 = 1$. Applying this formula naively to the solutions in (2), i.e. without allowing a dependence of the harmonic function on the internal coordinates via a periodic array, we find

$$A = \left(\int_{S_2} R^2(r) d\Omega\right)_{r=0} = \left(\int_{S_2} \sqrt{H^{\frac{2}{1+a^2}} r^2} d\Omega\right)_{r=0} \to \left(\omega_2 \, r_h^{\frac{2}{1+a^2}} \, r^{\frac{a^2}{1+a^2}}\right)_{r=0} \ , \tag{16}$$

where $R(r)$ is the radius of the sphere for fixed value of r and $\omega_2 = 4\pi$ is the 2d unit-sphere volume. Thus, one gets only for the RN solution ($a = 0$) a non-vanishing Bekenstein-Hawking entropy: $S = 2\pi Q^2$ for $r_h = \sqrt{2}\,Q$. Since on the other side the entropy has the statistical interpretation of counting all possible states this is not the desired result.

As discussed e.g. in [3], it seems strange, that only the case of 4 non-vanishing charges yields a non-vanishing entropy, whereas for all other cases one gets zero. However, one can argue that this shortcoming is simply a consequence of an ill–defined perturbation theory. Either the theory is in the strong coupling regime (for the magnetic solutions) indicating a failure of the string perturbation theory (large string coupling constant g_s) or there is a curvature singularity in the string metric (electric solutions) indicating a break down of the low energy limit (large α' corrections). Both perturbation series are under control for the RN solution after generalizing it to an independent electric (Q) and a magnetic (P) charge, which is equivalent to two independent harmonic functions. Then, on the horizon we have for the string coupling constant $g_s^2 = e^{2\phi} \sim (\frac{P}{Q})$ and the string metric at the horizon is regularized by the magnetic charge, which effectively corresponds to a renormalization of α' [2] $\alpha' \to \frac{\alpha'^2}{P^2}$. So, assuming that $Q \gg P \gg 1$, i.e. $\alpha', g_s \ll 1$ the theory is well defined even on the horizon. Here, we argue that similar to the RN case one can keep also for the other cases both perturbation expansions under control by allowing the maximal possible dependence of the harmonic functions on the transversal coordinates and assuming that the charges are large. As explained for the electric $a = 1/\sqrt{3}$ case this has the consequence that the solution decompactifies to its higher-dimensional origin. The asymptotic geometry is given by (14) and as in 4 dimensions the entropy is defined by integrating over the spherical part. However, this gives not the total entropy, but the entropy per unit world volume. Note that the AdS part in (14) consists of the radius and world volume, which are kept fixed in this calculation (no integration over these coordinates). To consider this quantity is also motivated by the suggestion of [15] that the entropy is given by the minima of the susy central charge, which in turn is equal to the mass per unit world volume (Bogomol'nyi bound). For the total entropy we have to keep in mind that on the one hand the decompactified branes are infinitely extended objects and on the other hand the world volume components of the target space metric vanish on the horizon, i.e. one has to define a suitable limit. Keeping the brane compactified, i.e. wrapped around a torus results in a vanishing total entropy. One can get a non-trivial result if one goes to the non-extremal case. For the D-3-brane this has been investigated in [17].

Let us now compare the different entropy contributions. First, we note that the radius of all horizons is given by r_h, which can be expressed by the electric and magnetic charges. Since the 4 dimensional solution can be electric as well as magnetic, let us distinguish between both charges. Then for $a = 0, 1, \sqrt{3}$, after integrating over the different spherical parts S_k (i.e. $k = 2 + a^2$) the entropy per unit world volume can be written as

$$\begin{array}{c} S = \frac{A}{4G_k} = \frac{1}{4G_k} \int_{S_k} (r_h)^k d\Omega_k = \pi (r_h)^k \\ \text{with:} \quad (r_h)^2 = 4\sqrt{\left((\vec{n} + \tfrac{1}{2}\vec{Q})L(\vec{n} + \tfrac{1}{2}\vec{Q})\right)\left((\vec{p} + \tfrac{1}{2}\vec{P})L(\vec{p} + \tfrac{1}{2}\vec{P})\right)}\ . \end{array} \tag{17}$$

where L is the metric in the $O(d,d)$ space[4] and $\vec{n}$ and $\vec{p}$ are arbitrary unit vectors ($\vec{n}L\vec{n} =$

[4]In this formula we have already included the different ways of embedding the intersection into the 10d space, which causes this $O(d,d)$ structure.

$\vec{p}L\vec{p} = 1$). With G_k we take into account that the Newton constant has to be rescaled when one compares expressions in different dimensions (see e.g. [16]). In our normalization in 4 dimensions we have $G_4 = 1$.

From the black holes in (2) we do not see that some of these solutions have a hidden part of the horizon. Instead, to measure the area of the horizon one could draw an S_2 around the origin and shrink the radius untill we reach the horizon. By this procedure one would define a common S_2 cut through all horizons. Integrating over this S_2 only (i.e. $k = 2$ in (17)) and assuming that $\vec{n} \cdot \vec{Q} = \vec{p} \cdot \vec{P} = 0$ yields the entropy formula suggested by [18]. In this reference one can find an explanation of the case where we have no charges at all. In this case, the term $S_c(Q = P = 0) = 2\sqrt{\frac{\pi^2}{6} \times 24}$ is related to the world sheet zero point energy of 24 transversal oscillators. S_c indicates that the entropy has been obtained by integrating only over the common S_2 cut. If we take now the case of $d = 2$: $\vec{Q} = (Q_1, 0, Q_2, 0)$, $\vec{P} = (0, P_1, 0, P_2)$, $\vec{n} = (0, 0, 1, 0)$, $\vec{p} = (0, 0, 0, 1)$ and all non-vanishing charges large (to keep the 4d perturbation expansions under control), the separate cases are:

(i) $a = 0$: $S_c = 2\pi\sqrt{Q_1 Q_2 P_1 P_2}$,

(ii) $a = 1$: $S_c = 4\pi\sqrt{Q_1 Q_2 / 2}$ ($P_1 = P_2 = 0$, electric case),

(iii) $a = \sqrt{3}$: $S_c = 4\pi\sqrt{Q_1}$ ($Q_2 = P_1 = P_2 = 0$, electric case)

or identifying the charges (or harmonic functions) we get

$$S_c = 2\pi\sqrt{1 + a^2}\, |Q|^{\frac{2}{1+a^2}} \ . \tag{18}$$

For the $a = 1, \sqrt{3}$ cases we considered only the electric part. Of course, via S-duality in 4 dimensions every electric solution has its magnetic analogue with the charges $\vec{P} = -L\vec{Q}$. This solution gives the magnetic part of the entropy. Note that the charge vectors are perpendicular to each other. This is a result of the fact that all $U(1)$'s are related to different directions in the internal space. In [2], in the case of $a = 0$, it has been argued that the other cases with arbitrary charge vectors (and all charges large) correspond to a non-vanishing axion in 4 dimensions. In this case the area formula has to include a correction term ($\sim \vec{P}L\vec{Q}$). Since the axion is related to a NS charge and we are considering only RR intersections it is natural that we do not get this term.

In our approach the case $a = 1/\sqrt{3}$ is special. The electric case yields, integrating over S_3 an entropy density $S \sim \sqrt{Q^3}$ whereas the magnetic case leads, after integrating over S_2, to $S \sim P^2$. Recently, many authors have investigated the electric case (see e.g. [19]). However, it does not fit into our entropy formula (we have 3 intersecting branes in this case). After restricting to the common S_2 cut we have $S \sim Q$ for the electric case and $\sim P^2$ for the magnetic case. To get the right power of charges, it seems that we have to take the average of the electric and magnetic contributions. Note that the $a = 1/\sqrt{3}$ BH is also special in the sense that it cannot be expressed by D-3-branes only. In order to get a non-singular result we need to include the wave or Taub-NUT soliton. Thus, we can conclude that the formula (17) describes the entropy for all BH's that can be expressed in a non-singular way by D-3-branes only.

We have considered only the non-singular intersections yielding the BH's in (2). Of course, there are other higher–dimensional solutions yielding the same black holes after compactification to $d = 4$. But all these solutions remain singular or strongly coupled near the horizon resulting in a vanishing entropy. Therefore, from all possible states only the discussed intersections contribute to the entropy counting. These non-singular states are preferred by the system.

4. Conclusions

In this letter we have discussed extremal 4d Maxwell/scalar black hole solutions with scalar coupling parameter a near the horizon. Only the $a = 0$ black hole has a non-vanishing Bekenstein-Hawking entropy. All black hole solutions that include a scalar field coupling have usually vanishing entropy. This is a puzzle, since the entropy counts the states and it should be possible to have different states like, e.g. , pure magnetic or pure electric configurations. On the other side we know that all these solutions appear as the compactification of higher–dimensional solutions. There are many possibilities to describe these solutions as intersections of p-branes in 10 dimensions. But for every black hole type there exist just one possibility which is non-singular and for which one can define a non-vanishing Bekenstein-Hawking entropy: (i) the 10d type IIB 3-brane for $a = \sqrt{3}$, (ii) the 6d type IIB 1-brane for $a = 1$ and (iii) 5d RN solution for the $a = 1/\sqrt{3}$ solution. The first 2 cases can be understood as intersections of 3-branes only. The last case, however, requires an additional wave or Taub-NUT soliton for the intersection. Compactifying these objects over periodic arrays results in an effective higher–dimensional solution near the horizon. As a consequence, the singularities disappear [4] yielding a non-trivial entropy. The different cases have different spherical symmetry near the horizon. Integrating over the horizon we have given an entropy formula (17) that covers the cases of pure D-3-brane intersections ($a = 0, 1, \sqrt{3}$). This generalizes the entropy formula given in [18] to the case of a single D-3-brane.

Acknowledgements

The work of K.B. is suported by the DFG. The work of E.B. has been made possible by a fellowship of the Royal Netherlands Academy of Arts and Sciences (KNAW).

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